Singularity Structure in Veneziano's Model

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We consider the structure of the cosmological singularity in Veneziano's inflationary model. The problem of choosing initial data in the model is shown to be unsolved — the spacetime in the asymptotically flat limit can be filled with an arbitrary number of gravitational and scalar field quanta. As a result, the universe acquires a domain structure near the singularity, with an anisotropic expansion of its own being realized in each domain.

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I. INTRODUCTION

An effective inflationary model is known to be very difficult to construct in the low-energy approximation of the string theory [1]. The effective potentials of the scalar field responsible for the inflationary pattern of cosmological dynamics that arise in attempting to solve the problem do not ensure the satisfaction of the slow-roll condition $|\dot{H}| \ll H^2$. Thus, the Friedman decelerating expansion is typical of this effective gravitational theory even in the presence of a scalar field. Actually, this implies that the problem cannot be solved by a brute-force method: new ideas based on the nontrivial low-energy spectrum of the string theory or the non-perturbative effects arising in this theory should be invoked to obtain an inflationary scenario in terms of the string theory. One might probably expect the field dynamics at the inflationary phase to be also nontrivial. The appearance of Veneziano's paper [2] may be considered to be the birth time of one of these nontrivial scenarios. The basic idea of this work is as follows. Let us consider the low-energy effective action of an

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arbitrary theory of closed superstrings [5], with our analysis being restricted to the field problem with zero vacuum averages of the fermion fields, the $R - R -$ sector fields and the antisymmetric field $B_{\mu\nu}$. The sector left after this projection has the same structure for any superstring theory [18], and the corresponding effective action (in dimensionless units) is

$$S = -\int \sqrt{-g}d^{d+1}xe^{-\phi}\left(R + g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) \tag{1}$$

Here, d is the dimension of space. We will seek a spatially homogeneous classical solution to the equations of motion derived by varying this action. To this end, as usual, we should set

$$g_{\mu\nu} = (1, -a^2(t)\delta_{ij}), \tag{2}$$

$$\phi = \phi(t) \tag{3}$$

Substituting (2) and (3) into (1), we easily find that

$$S = -\int d^{d+1}xa^de^{-\phi}\left(\dot{\phi}^2 - 2dH\dot{\phi} + d(d-1)H^2\right) \tag{4}$$

As can be easily seen, this action is invariant relative to the field transformations

$$a(t) \rightarrow \frac{1}{a(t)}, \tag{5}$$

$$\phi \rightarrow \phi - 2d\ln(a), \tag{6}$$

$$t \rightarrow -t \tag{7}$$

Therefore, the equations of motion following from (4) have a classical solution that describes the accelerating expansion corresponding to inflation at $-\infty < t < -0$ and the Friedman decelerating expansion at $+0 < t < +\infty$:

$$a_+(t) = (t/t_0)^{1/\sqrt{d}}, \phi = \left(\sqrt{d} - 1\right)\ln\left(\frac{t}{t_0}\right), t > 0; \tag{8}$$

$$a_-(-t) = (-t/t_0)^{-1/\sqrt{d}}, \phi = -\left(\sqrt{d} + 1\right)\ln\left(-\frac{t}{t_0}\right), t < 0 \tag{9}$$

The physical meaning of this solution is as follows. The universe is initially an asymptotically flat world in the sense that the Riemann tensor components tend to zero as $t \rightarrow -\infty$. In addition, this world is absolutely cold — it contains no clustered matter. Starting from this maximally symmetric state, the universe undergoes superinflation (the prefix "super" implies that $\dot{H} > 0$) at $-\infty < t < -0$ and gives way to decelerating Friedman expansion

at $t > 0$ which corresponds to the transition from the superinflationary branch to the dual one. Significantly, as $t \to -0$, the spacetime curvature tends to infinity. Therefore, sooner or later, we will go outside the validity range for the low-energy approximation of the string theory and action (1) in our solution. The invariance of action (4) with respect to transformations (5) – (7) was called SF duality [19]. If it holds not only for the low-energy approximation of the string theory but also for the total non-perturbative Green–Schwartz sigma model, then a solution of type (8) and (9) actually corresponds to the saddle point in the complete field–theoretic problem; i.e., it describes the cosmological dynamics in this theory. However, since the SF duality itself is an essentially non-perturbative effect. That is why we cannot ascertain whether it exists in the non-perturbative string theory restricting ourselves by analysis of low-energy dynamics. Gasperini and Veneziano [3] argued that the non-perturbative SF duality does takes place. The most interesting and critical (from the viewpoint of the scenario) point on the time axis is $t = 0$ at which the cosmological singularity is reached [20]. Our objective is to ascertain the pattern of field dynamics near the singularity and understand whether an accounting for the fluctuations of the classical trajectory (8), (9) leads to a general softening of the singularity in the low-energy approximation.

II. AN ANISOTROPIC SOLUTION IN VENEZIANO'S MODEL

The equations of motion that follow from the four-dimensional theory with the action

$$S_{(S)} = -\frac{1}{\lambda_s^2} \int \sqrt{-g} d^4 x e^{-\phi} \left(R + (\partial\phi)^2 \right), \tag{10}$$

(here, we introduced the constant λ_s to reduce the action to dimensionless form) are

$$- D_\mu D_\nu \phi + \frac{1}{2} g_{\mu\nu} (D\phi)^2 = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{11}$$

$$D_\mu D^\mu \phi = (\partial\phi)^2. \tag{12}$$

Analysis of these equations can be greatly simplified if we note that the theory described by action (10) is conformally equivalent to the general theory of relativity with the scalar field

$$S_{(E)} = -\frac{1}{\lambda_s^2} \int \sqrt{-g} d^4 x \left(R - \frac{1}{2} (\partial\phi)^2 \right), \tag{13}$$

Indeed, if the metric $g_{\mu\nu(E)}$ is the solution of the Einstein equations, then $g_{\mu\nu(S)} = g_{\mu\nu(E)} e^{\phi}$ is the solution of Eqs. (11) and (12). The reverse is also true (below, we call $g_{\mu\nu(E)}$ and $g_{\mu\nu(S)}$ the metrics in the Einstein and string frames, respectively).

Let us try to go beyond the scope of the homogeneous problem and to construct an exact solution that corresponds to a classical strong gravitational wave propagating against a homogeneous background and strong inhomogeneous scalar field perturbations in the Einstein frame. Since, in general, this solution, clearly, cannot be found, we restrict our analysis to the quasi-two- dimensional problem. Let all metric components and the scalar field depend on the coordinates t and x alone. This problem can be solved completely (see, e.g., [6] , [7]), and the answer is the axisymmetric Einstein – Rozen metric.

We seek the corresponding solution in the form

$$ds^2_{(E)} = e^{2A}dt^2 - e^{2C}dx^2 - e^{2B}(e^\gamma dy^2 + e^{-\gamma}dz^2), \tag{14}$$

where A, B, C, γ and ϕ are functions of t and x alone. Since the Einstein equations are invariant relative to the gauge transformations $\tilde{t} = \tilde{t}(t,x)$ and $\tilde{x} = \tilde{x}(t,x)$, we may set $g_{00} = -g_{11}, g_{01} = 0$, i.e., $A(t,x) = C(t,x)$.

The Einstein equations impose the following constraints on the functions A, B, γ and ϕ:

$$A'' - \ddot{A} - 2\ddot{B} + 2\dot{A}\dot{B} + 2A'B' - 2(\dot{B})^2 - \frac{(\dot{\gamma})^2}{2} = \frac{1}{2}(\dot{\phi})^2, \tag{15}$$

$$-2\dot{B}' + 2A'\dot{B} + 2B'\dot{A} - 2B'\dot{B} - \frac{\dot{\gamma}\gamma'}{2} = \frac{1}{2}\phi'\dot{\phi}, \tag{16}$$

$$\ddot{A} - A'' - 2B'' + 2A'B' + 2\dot{A}\dot{B} - 2(B')^2 - \frac{(\gamma')^2}{2} = \frac{1}{2}(\phi')^2, \tag{17}$$

$$\ddot{B} + 2(\dot{B})^2 = B'' + 2(B')^2 \tag{18}$$

$$\ddot{\gamma} + 2\dot{B}\dot{\gamma} = \gamma'' + 2B'\gamma' \tag{19}$$

Finally, the equation of motion for the field ϕ appears as

$$\ddot{\phi} + 2\dot{B}\dot{\phi} = \phi'' + 2B'\phi' \tag{20}$$

We can easily find from (18) that

$$B = \frac{1}{2}\ln(f_1(\xi) + f_2(\eta)), \tag{21}$$

where $\xi = t - x$, $\eta = t + x$ and $f_{1,2}$ re arbitrary functions of their argument. Let us now use the remaining gauge invariance with respect to the transformations $\tilde{\tilde{\xi}} = H_1(\xi), \tilde{\eta} = H_2(\eta)$ and set $B = \frac{1}{2}\ln\left(-\frac{t}{t_i}\right)$. Equations (19) and (20) can then be easily solved:

$$\phi = \psi\ln(-t/t_i) + \sum_k (c_{1k}J_0(kt) + c_{2k}N_0(kt))e^{ikx} + \text{c.c.} \tag{22}$$

$$\gamma = \beta \ln(-t/t_i) + \sum_k (c_{3k} J_0(kt) + c_{4k} N_0(kt)) e^{ikx} + \text{c.c.} \tag{23}$$

An expression for $A(t,x)$ can be derived by using the equations

$$\dot{A} = \frac{t}{4}((\phi')^2 + (\dot{\phi})^2 + (\gamma')^2 + (\dot{\gamma})^2 - \frac{1}{t^2}) \tag{24}$$

$$A' = \frac{t}{2}(\phi'\dot{\phi} + \gamma'\dot{\gamma}), \tag{25}$$

It is convenient to separate the function $A(t,x)$ into the homogeneous and inhomogeneous parts. The former can be easily derived from Eq. (24):

$$A_{\text{hom}} = \frac{1}{4}(\psi^2 + \beta^2 - 1)\ln(-t/t_i) + \frac{1}{4}\sum_k \left(c_{1k}c_{1k}^+ \frac{(kt)^2}{2}(J_1^2(-kt) - \right.$$

$$J_0(-kt)J_2(-kt)) + (c_{2k}c_{1k}^+ + c_{1k}c_{2k}^+)\frac{(kt)^2}{4}(2J_1(-kt)N_1(-kt) -$$

$$J_2(-kt)N_0(-kt) - J_0(-kt)N_2(-kt)) +$$

$$\left. c_{2k}c_{2k}^+ \frac{(kt)^2}{2}(N_1^2(-kt) - N_0(-kt)N_2(-kt)) \right) +$$

$$\frac{1}{4}\sum_k (c_{1k}c_{1k}^+ \frac{(kt)^2}{2}(J_0^2(-kt) + J_1^2(-kt)) +$$

$$c_{2k}c_{2k}^+ \frac{(kt)^2}{2}(J_0^2(-kt) - J_1^2(-kt)) +$$

$$(c_{1k}c_{2k}^+ + c_{2k}c_{1k}^+)\frac{(kt)^2}{4}(2J_0(-kt)N_0(-kt) - J_1(-kt)N_{-1}(-kt) -$$

$$J_{-1}(-kt)N_1(-kt))) + (1,2) \to (3,4) \tag{26}$$

The inhomogeneous contribution to the function $A(t,x)$ is easier to determine from Eq. (25):

$$A_{\text{inh}} = \psi \sum_k (c_{1k}J_0(-kt) + c_{2k}N_0(-kt)) e^{ikx} +$$

$$\sum_{k,l} \frac{klt}{k+l} e^{i(k+l)x}(c_{1k}J_1(-kt) + c_{2k}N_1(-kt))(c_{1l}J_0(-lt) + c_{2l}N_0(lt) +$$

$$\sum_{k,l,k\neq l} \frac{klt}{k-l}(c_{1k}J_1(-kt) + c_{2k}N_1(-kt))(c_{1l}J_0(-lt) + c_{2l}N_0(-lt)) + \text{c.c.} +$$

$$(1,2,\psi) \to (3,4,\beta) \tag{27}$$

Let us first consider the homogeneous limit ($c_{\alpha k} = 0$, $\forall \alpha, k$). In the string frame, the spacetime metric is

$$ds^2 = \left(-\frac{t}{t_i}\right)^{\frac{1}{2}(\psi^2+\beta^2-1)+\psi}(dt^2 - dx^2) - \left(-\frac{t}{t_i}\right)^{1+\psi+\beta}dy^2 -$$

$$\left(-\frac{t}{t_i}\right)^{1+\psi-\beta} dz^2 \tag{28}$$

The scalar curvature corresponding to this metric is

$$R = -\frac{\psi^2}{t^2}\left(-\frac{t}{t_i}\right)^{1/2(1-\beta^2-2\psi-\psi^2)} = -\frac{\psi^2}{t_i^2}\left(-\frac{t_i}{t}\right)^{1/2(2+\beta^2+(\psi+1)^2)} \tag{29}$$

It tends to zero as $t \to -\infty$ for any point in parametric space (ψ, β). If $\beta = 0$ and $\psi^2 = 3$, then the spacetime (the background spacetime in the inhomogeneous problem) is isotropic. In this case, metric (14) is identical to Veneziano solution, which is asymptotically equivalent to Minkowski flat universe for $t \to -\infty$. Below, precisely this choice of parameters will be of particular interest to us.

III.   THE LIMIT OF ASYMPTOTICALLY FLAT SPACETIME. QUANTIZATION

Below, we will see that the modes in expressions (22) and (23) can be interpreted as dilatons and gravitons propagating against the background of curved spacetime. The initial conditions are chosen in the following way: we specify a sufficiently large time t_i (this is the time scale which appears in (22), (23), and(26)) and discard all the modes that do not satisfy the condition $k|t_i| \gg 1$; i.e., we neglect the modes with a wavelength larger than the cosmological horizon in the initial state. However, the existence of these modes is in conflict with causality, unless the initial state itself arose from inflation. Below, by the limit $t \to -\infty$, we mean all t such that $|t| \gg |t_i|$. In this case, the following asymptotics is possible for all the modes without exception:

$$J_0(-kt) \approx \sqrt{\frac{2}{\pi(-kt)}} \cos\left(-kt - \pi/4\right),$$

$$N_0(-kt) \approx \sqrt{\frac{2}{\pi(-kt)}} \sin\left(-kt - \pi/4\right) \tag{30}$$

It thus follows that the "correct" [21] modes are

$$\frac{1}{2}e^{i\pi/4}H_0^{(1)}(-kt)e^{-ikx}$$

and

$$\frac{1}{2}e^{i\pi/4}H_0^{(1)+}(-kt)e^{ikx}.$$

which correspond to the substitutions

$$b_{1k} = e^{i\pi/4}(c_{1k} - ic_{2k}), b_{2k} = e^{i\pi/4}(c_{1k}^+ - ic_{2k}^+) \qquad (31)$$

Let us now turn to quantization. To properly normalize the modes $u_k = \frac{1}{2}e^{i\pi/4}H_0^{(1)}(-kt)e^{ikx}$ we calculate the commutator $[b_k, b_{k'}^+]$. Assuming that

$$[\phi(t,x), \phi(t,x')] = 0, [\pi(t,x), \pi(t,x')] = 0, \qquad (32)$$

$$[\phi(t,x), \pi(t,x')] = -\frac{i}{L^2}\delta(x-x'), \qquad (33)$$

where L is the infrared cut-off (linear size of the system under consideration) and

$$\pi(t,x) = \frac{\delta\sqrt{-g}L}{\delta\dot{\phi}} = -\frac{2}{\lambda_s^2}\sqrt{-g}e^{-\phi}g^{00}\dot{\phi},$$

we can easily find that

$$[b_k, b_{k'}^+] = \frac{\pi\lambda_s^2 t_i}{2L^3}\delta_{kk'} \qquad (34)$$

and the correctly (half-quantum) normalized modes for $t \to -\infty$ are

$$u_k = \sqrt{\frac{-\lambda_s^2 t_i}{L^3 kt}}e^{-ik(x-t)}.$$

To physically interpret these quantum modes, we calculate $\langle T_{00}\rangle$. In the Einstein equations, the $(0,0)$–component is

$$4\dot{A}\dot{B} - 2\ddot{B} - 2\dot{B}^2 = \frac{1}{2}((\dot{\phi})^2 + (\phi')^2 + (\dot{\gamma})^2 + (\gamma')^2) = T_{00} \qquad (35)$$

(we carried over γ's to the right and now interpret this part of the metric as the contribution of gravitons). If the paired correlators are assumed (in this case, it does not matter whether we consider the amplitudes of the modes $b_{\alpha k}$ as classical but randomly distributed Gaussian variables or as operators) to be

$$\langle b_{\alpha k}b_{\beta l}^+\rangle = n_\alpha(k)\delta_{\alpha\beta}\delta_{kl}, \langle b_{\alpha k}b_{\beta l}\rangle = 0 \qquad (36)$$

$$n_1(k) = n_2(k), n_3(k) = n_4(k) \qquad (37)$$

(the physical meaning of this condition is that the flux of rightward-propagating quanta is equal to the flux of leftward-propagating quanta), then

$$\langle T_{00}\rangle = \langle \hat{H}\rangle = \sum_k \frac{\lambda_s^2 k t_i}{L^3 t}(n_1(k) + n_3(k)) \qquad (38)$$

Thus, we see that for $t \to -\infty$, the asymptotically flat world is filled with gravitational and scalar field quanta, which are naturally called gravitons and dilatons, respectively. Their energy density tends to zero as $t \to -\infty$ (which is nothing than usual redshift) and can not be neglected. The cosmological solutions with zero occupation numbers have a zero measure in the functional space of the field-theoretic problem and, in this sense, are untypical. Thus, there is an infinite arbitrariness in choosing the initial state for Veneziano model, and the problem of initial data is yet to be solved (see in this context [8]).

IV. SPACETIME STRUCTURE NEAR THE SINGULARITY

In the limit $t \to -0$, the spacetime metric in the string frame is asymptotically equivalent to the Kasner metric with the indices

$$p_1 = \frac{1/2((\beta + \beta_1)^2 + (\psi + \psi_1)^2 - 1) + \psi + \psi_1}{1/2((\beta + \beta_1)^2 + (\psi + \psi_1)^2 + 3) + \psi + \psi_1}, \tag{39}$$

$$p_2 = \frac{1 + \psi + \psi_1 + \beta + \beta_1}{1/2((\beta + \beta_1)^2 + (\psi + \psi_1)^2 + 3) + \psi + \psi_1}, \tag{40}$$

$$p_3 = \frac{1 + \psi + \psi_1 - \beta - \beta_1}{1/2((\beta + \beta_1)^2 + (\psi + \psi_1)^2 + 3) + \psi + \psi_1}, \tag{41}$$

where

$$\psi_1 = \sum_k \frac{2}{\pi} N \left(e^{i\frac{\pi}{4}} (b_{1k} e^{ikx} + b_{2k} e^{-ikx}) + \text{c.c.} \right) \tag{42}$$

$$\beta_1 = \sum_k \frac{2}{\pi} N \left(e^{i\frac{\pi}{4}} (b_{3k} e^{ikx} + b_{4k} e^{-ikx}) + \text{c.c.} \right), \tag{43}$$

and $N = \sqrt{\frac{\pi \lambda_s^2 t_i}{2L^3}}$ is the normalization constant for the modes calculated in the preceding section. The contribution to the action of dilaton ϕ from the part of the classical trajectory (22) in the vicinity of the cosmological singularity ΔS_{sing} logarithmically diverges — it is proportional to $\ln\left(\frac{\Lambda}{t_0}\right)$, where Λ is the beginning of the Kasner epoch, and $t_0 \to 0$ is some small time scale. At the same time, the remaining contribution to the action is finite. This can be interpreted as the destruction of quantum coherence [9] between the modes c_{1k} and c_{2k} (c_{3k} and c_{4k}) for which the condition $|kt| \ll 1$ is satisfied. Because of this destruction, the modes with a wavelength larger than the cosmological horizon freeze — their amplitudes may be considered to be classical, randomly distributed variables rather than operators [22]. These modes contribute to the Kasner indices. Thus, the spacetime structure becomes

stochastic as $t \to -0$, and it makes sense to discuss the behavior of the various correlation functions of the Kasner indices. Below, we disregard the time-independent contributions to the metric, because they bear no relation to the character of the metric singularity for $t \to -0$.

Technically, it is more convenient not to pass to world time but work with a metric of the form $ds^2 = t^{q_1}(dt^2 - dx^2) - t^{q_2}dy^2 - t^{q_3}dz^2$. The quantities q_1, q_2 and q_3 are related to the Kasner indices by

$$q_1 = \frac{2p_1}{1-p_1}, q_2 = \frac{2p_2}{1-p_1}, q_3 = \frac{2p_3}{1-p_1} \tag{44}$$

Since the identity $p_1^2 + p_2^2 + p_3^2 = 1$ takes place in the string frame, the following relation is also valid:

$$q_2^2 + q_3^2 = 4(q_1 + 1) \tag{45}$$

he stochastic spacetime structure can be completely determined by calculating the distribution function $F(\lambda, \mu) = \langle \delta(q_2 - \lambda)\delta(q_3 - \mu)\rangle = \int dxdy e^{-i(\lambda x + \mu y)}\langle e^{i(q_2 x + q_3 y)}\rangle$. Since q_1 can be unambiguously determined from the known quantities q_2 and q_3, this distribution function allows expressions for any correlators of the indices q_i to be derived. After simple calculations, we obtain

$$F(\lambda, \mu) = \frac{\pi}{\sqrt{N_1 N_3}}\exp\left(-\frac{(1 + \psi - \frac{\lambda+\mu}{2})^2}{2N_1} - \frac{(\beta - \frac{\lambda-\mu}{2})^2}{2N_3}\right) \tag{46}$$

where $N_1 = \sum_k \frac{n_{1k}N^2}{\pi^2}$ and $N_3 = \sum_k \frac{n_{3k}N^2}{\pi^2}$.

This function characterizes the spacetime "in the infinitely small". Since the problem is translationally invariant, all points in space are equal in rights: the locally measured Kasner indices are the random variables described by the distribution function (46). However, the probability that the Kasner indices are small at a given point and large at a infinitely close point approaches zero. The nonlocal correlation properties of the Kasner indices are specified in part by the two-point correlation functions [23]

$$\langle q_2(x)q_2(x')\rangle = (1 + \psi + \beta)^2 + \sum_k \frac{16N^2(n_{1k} + n_{3k})}{\pi^2}\cos k(x - x'), \tag{47}$$

$$\langle q_3(x)q_3(x')\rangle = (1 + \psi - \beta)^2 + \sum_k \frac{16N^2(n_{1k} + n_{3k})}{\pi^2}\cos k(x - x'), \tag{48}$$

$$\langle q_2(x)q_3(x')\rangle = (1 + \psi)^2 - \beta^2 + \sum_k \frac{16N^2(n_{1k} - n_{3k})}{\pi^2}\cos k(x - x') \tag{49}$$

We can see that the correlation is oscillatory in pattern, which is an artifact of specifying the initial conditions for $t \to -\infty$.

In conclusion, let us consider the transition possibility from the superinflationary expansion of the universe to its contraction as the cosmological singularity is approached.

If the spacetime metric is isotropic and uniform, then we say that the universe undergoes superinflationary expansion when the condition $\dot{H} = \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 > 0$, is satisfied, where a is the scale factor. If the latter changes with time as $a = (-t)^p$, then the inequality is satisfied for $p < 0$. This criterion can be generalized to the anisotropic case in two ways (as we will see below, the results depend only slightly on the choice of a criterion):

(i) by introducing not one but three scale factors and requiring the satisfaction of the inequalities $p_1 < 0$, $p_2 < 0$, and $p_3 < 0$;

(ii) by requiring the satisfaction of the inequality $p_1 + p_2 + p_3 < 0$, which characterizes the behavior of the comoving 4-volume element.

Since the spacetime acquires a stochastic structure that is locally characterized by distribution (46) as the singularity is approached, there is a nonzero probability that superinflation will stop and will give way to contraction. It is easy to see that, according to criterion (i), the inverse (in a sense) quantity — the probability that superinflation continues until the fall of the universe to a singularity [24] — is

$$w(\forall p_i < 0) = w(\forall q_i < 0) = \frac{1}{2\pi\sqrt{N_1 N_3}} \int_0^{\sqrt{2}} \rho d\rho \times$$

$$\int_{3\pi/4}^{5\pi/4} d\phi \exp\left(-\frac{(1 + \psi - \rho\cos\phi)^2}{2N_1} - \frac{(\beta - \rho\cos\phi)^2}{2N_3}\right) \tag{50}$$

In the physically interesting case with $N_1, N_3 \to \infty$ and $\psi, \beta \sim 1$ we have

$$w(\forall p_i < 0) \sim \frac{1}{8\sqrt{N_1 N_3}} \tag{51}$$

The probability that superinflationary expansion will give way to contraction in all directions in the same limit behaves as

$$w(\forall p_i > 0) \sim \frac{1}{\pi}\arctan\sqrt{\frac{N_1}{N_3}} - \frac{1}{8\sqrt{N_1 N_3}} \tag{52}$$

If, however, we proceed from criterion (ii), then the probability of maximum superinflation duration is

$$w(\sum p_i < 0) = \frac{1}{2\pi\sqrt{N_1 N_3}} \int_0^{\sqrt{6}} \rho d\rho \int_0^{2\pi} d\phi \exp\left(-\frac{(3 + \psi - \rho\cos\phi)^2}{2N_1}\right)$$
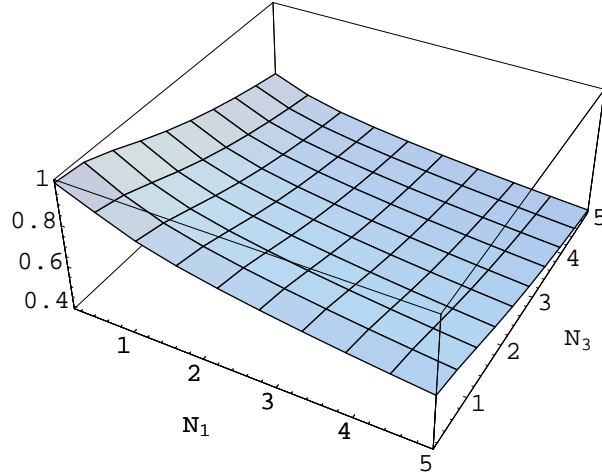
FIG. 1: Behavior of the probability of maximum superinflation duration, $w(\sum p_i < 0)$, as a function of N_1 and N_3 for $\psi = -\sqrt{3}, \beta = 0$ at relatively low graviton and dilaton densities in the initial state.

$$-\frac{(\beta - \rho \sin \phi)^2}{2N_3}\Bigg) \sim \frac{3}{\sqrt{N_1 N_3}}(N_1, N_3 \to \infty; \psi, \beta \sim 1) \qquad (53)$$

Since asymptotics (50) and (53) are virtually independent of the choice of a superinflation duration criterion, we can say that superinflationary expansion necessarily gives way to contraction as the cosmological singularity is approached.

V. CONCLUSIONS

We have considered the structure of the cosmological singularity in Veneziano's model. As we showed, the problem of uniqueness in choosing the initial conditions in Veneziano's scenario is yet to be solved — the asymptotically flat world that corresponds to the initial state in the scenario can be filled with gravitational and scalar field quanta. In order to understand what influence any variations in initial data have on the spacetime structure near the singularity, we constructed an exact solution that described the gravitational and scalar field quanta propagating against the background of asymptotically flat spacetime in the limit $t \to -\infty$.

The first essentially new effect that we have found is the hard imprint of initial conditions on the structure of spacetime near the singularity. Primordial quantum fluctuations
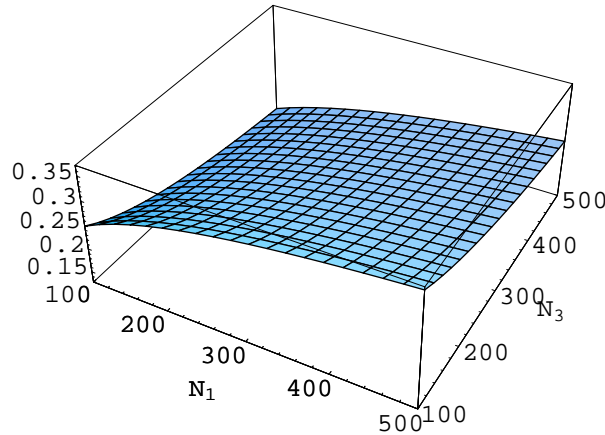
FIG. 2: Behavior of the transition probability from superinflationary expansion to contraction in all directions, $w(\forall p_i > 0)$, as a function of N_1 and N_3 for $\psi = -\sqrt{3}, \beta = 0$ and high energy densities of gravitons N_1 and dilatons N_3 in the initial state.

of gravitational and matter fields cause the spacetime to acquire domain structure, this probabilistic domain structure being completely determined by the correlations of fields in the initial state for superinflation. Indeed, though we deal with a strong gravitational background near the singularity, effects of particle creation are not important for the large-scale structure [25] of spacetime at $t = 0$. Near the singularity the characteristic wavelength of created particles $\lambda \sim R^{-1/2}$ is much less then the typical size of domain $1/k$ — a constant defined by initial conditions for the scenario. Also, back-reaction of created particles can be neglected until we reach the string scale [26], where the low-energy approximation is no longer valid. That is why created particles are not important, and large-scale structure of spacetime near the singularity is completely described by initial conditions, i.e., number of particles in the initial state for superinflation.

Our another goal was to show once again that Kasner-like solution of Einstein equations found by Belinskii, Lifshitz and Khalatnikov [11] has the generic character near cosmological singularity. It is well known that if the matter sector of underlying theory contains only hydrodynamic types of matter, the approach toward singularity acquires chaotic features. Generally speaking, the situation is different, if we take into account such relativistic forms of matter as scalar field (see [12], [13]) — the generic solution of Einstein equations becomes to exhibit a non-oscillatory power-law behavior. Nevertheless, it has been shown [15], that

BKL chaos near singularity is resuscitated if the underlying theory of gravity is described by bosonic sector of low-energy string effective action. [27] It should be noted that chaotic structure of spacetime found in this note bears no relation to BKL chaos. The large-scale domain structure of spacetime near the singularity is completely determined by quantum initial conditions for the scenario. In a sense, this chaos is quantum, not classical one.

The physical reason for this chaotic behavior to appear is that the radius of the cosmological horizon $R^{-1/2}$ specifies the causal connectivity scale in the theory; accordingly, the quantum quantities can correlate only on scales smaller than $R^{-1/2}$. Freezing of amplitudes of initially quantum modes implies that an observer living at the Kasner epoch will always record the same Kasner indices, irrespective of the number of experiments that he or she carries out. Nevertheless, before the time for the Kasner asymptotics to become valid, we cannot predict with confidence what Kasner indices the observer will record — in this sense, the Kasner indices are random variables.

The second new effect that we have found in this note is the general softening of singularity due to the account of quantum initial conditions variety in the sense that at $t \to -0$ the spacetime dynamics tends to transition from superinflationary expansion to contraction. We believe that this fact can have a bearing on solving "the graceful exit" problem in Veneziano's string cosmology. Nevertheless, the regime in which this contraction is realized turns to be rather complicated.

Fig. 1 shows the behavior of the probability of maximum superinflation duration $w(\sum p_i < 0)$ as a function of total number of dilatons and gravitons in the initial state (N_1 and N_3, respectively) for $\psi = -\sqrt{3}$, $\beta = 0$, which corresponds to the absence of a seed anisotropy, i.e., Veneziano's universe. Interestingly, at moderately large N_1 and N_3, an increase in the graviton energy density with respect to the dilaton energy density generally causes this probability to decrease. Nevertheless, at large N_1 and N_3, the graviton and dilaton energy densities have the same weight from the viewpoint of their influence on the probability of maximum superinflation duration — an increase in the number of gravitons and dilatons in the initial state always causes this probability to decrease.

When considering the asymptotics of the transition probability from superinflation to contraction in all directions, $w(\forall p_i > 0)$ at large N_1 and N_2, we arrive at a similar picture (see Fig. 2): an increase in the number of gravitons in the initial state generally causes this probability to decrease, while an increase in the number of dilatons causes it to increase. This

implies that the statistical weight of the states describing anisotropic expansion (when there is expansion in two of the three directions in space and contraction in the third direction, etc.) becomes large; the higher the graviton energy density in the initial state, the larger this weight.

Finally, let us discuss the degree of generality of our results. To determine the structure of spacetime near the singularity, we actually used the simplified model describing quasi-two-dimensional dynamics of the four-dimensional theory. From the physical point of view this simplification means that initial conditions at asymptotically flat infinity $t \rightarrow -\infty$ present two colliding plane waves propagating along $t - x$ and $t + x$ axis (see [16], where the dependence between initial conditions for Veneziano's scenario and spacetime structure near the singularity has been determined from the classical point of view).

Let us complicate the problem a bit: imagine that initial conditions are described by a bath of plain waves propagating along a fixed surface. It turns out that this quasi-three-dimensional problem also can be solved exactly by means of inverse scattering method [17]. The corresponding quantum problem hardly can be solved, too, as well as the full four-dimensional problem, both classical and quantum one.

Nevertheless, since the classical and quantum dynamics of generic perturbations for Veneziano's scenario can be described in terms of colliding plain waves, one should think that in the case of general initial conditions the universe acquires a stochastic domain structure near the singularity, with proper anisotropic regime of expansion being realized in each domain.

Acknowledgments

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[18] Recall that there are four distinct theories of closed superstrings in ten dimensions: IIA, IIB, heterotic $SO(32)$ and heterotic E_8.

[19] SF stands for the scale factor.

[20] Possibly, the singularity is smoothed out in the non-perturbative string theory. The validity of this assumption is closely related to the possibility of solving "graceful exit" problem — the problem of passing through the singularity from superinflation to decelerating expansion[4] (see also [3]).

[21] I.e., those corresponding to the positive frequency solutions.

[22] Strictly speaking, this can be done if the effective occupation numbers satisfy the condition $\langle n_k \rangle \gg 1$. In this case, the non-commutativity of the coordinates and momenta may be disregarded. For a more detailed discussion, see [10].

[23] The remaining two-point correlation functions can be easily calculated by using Eq. (45), which relates the Kasner indices. We do not give the corresponding expressions here, because they are too cumbersome.

[24] This type of singularity is characterized by tending the effective gravitational constant to infinity.

[25] "Large scale" means any scale larger then $R^{-1/2}$, where R is the scalar curvature of spacetime.

[26] The exact criterion is $\lambda_s^2 e^\phi R^{-1} < 1$, see for details [3].

[27] See also [14] for the discussion of properties of Kasner-like solutions in the Horava–Witten cosmology.